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SPACE-TIME ADAPTIVE PROCESSING (STAP) PERFORMANCE IN NON-HOMOGENEOUS RADAR CLUTTER

ARCON Corporation

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13. ABSTRACT *(Maximum 200 words)*
Abstract report addresses the statistical analysis of the non-homogeneity detector (NHD) for non-Guassian interference scenarios. An important issue in STAP is that of homogeneity of training data. Non-homogeneity of the training data has a deleterious effect on STAP performance in that undernulled clutter significantly degrades detection and false alarm characteristics. Previous work in this area has proposed the use of non-homogeneity detector based on a generalized inner product (GIP). The unsuitability of the GIP based test for non-Guassian interference scenarios is noted. We present a new non-homogeneity detector for non-Guassian interference scenarios which can be modeled by a spherically invariant random process (SIRP). Our work includes a statistical analysis of the NHD for non-Guassian interference taking into account the fact that finite sample support is used for covariance estimation. In particular, exact theoretical expressions for the NHD test statistic PDF and the mean of a related test statistic are derived. We also note that the related test statistic admits a remarkably simple stochastic representation as a ratio of an F-distributed random variable and a beta-distributed loss factor. Based on this development, a formal goodness-of-fit test is presented. Performance analysis is carried out using simulated and measured data from the MCARM Program.

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Chapter 2

Executive Summary

This research effort represents our investigation on the problem of statistical analysis of the non-homogeneity detector for non-Gaussian interference scenarios. An important issue in STAP is that of homogeneity of training data. Non-homogeneity of the training data has a deleterious effect on STAP performance in that undernulled clutter significantly degrades detection and false alarm characteristics. Previous work in this area has proposed the use of a non-homogeneity detector (NHD) based on a generalized inner product (GIP). The unsuitability of the GIP based test for non-Gaussian interference scenarios is noted. We present a new nonhomogeneity detector for non-Gaussian interference scenarios which can be modeled by a spherically invariant random process (SIRP). Our work includes a statistical analysis of the NHD for non-Gaussian interference taking into account the fact that finite sample support is used for covariance estimation. In particular, exact theoretical expressions for the NHD test statistic PDF and the mean of a related test statistic are derived. We also note that the related test statistic admits a remarkably simple stochastic representation as a ratio of an F-distributed random variable and a beta-distributed loss factor. Based on this development, a formal goodness-of-fit test is presented. Performance analysis is carried out using simulated and measured data from the MCARM Program.

Chapter 3

Statistical Analysis of the Nonhomogeneity Detector for Non-Gaussian Interference Backgrounds

3.1 Introduction

An important issue in space-time adaptive processing (STAP) for radar target detection is the formation and inversion of the covariance matrix underlying the clutter/ interference. In practice, the unknown interference covariance matrix is estimated from a set of independent identically distributed (iid) target-free training data which is assumed to be representative of the interference statistics in a cell under test. Frequently, the training data is subject to contamination by discrete scatterers or interfering targets. In either event, the training data becomes nonhomogeneous. As a result, it is non representative of the interference in the test cell. Estimates of the covariance matrix from nonhomogeneous training data result in severely undernulled clutter. Consequently, CFAR and detection performance suffer. Significant performance improvement can be achieved by employing pre-processing to select representative training data.

The problem of target detection using improved training strategies has been considered in [1–8]. The impact of nonhomogeneity on STAP performance is considered in [8–11]. The works of [1–4, 8, 12] have addressed the use of the non-homogeneity detector (NHD) based on the generalized inner product (GIP) measure for STAP problems involving Gaussian interference scenarios. This work was extended significantly in [13,14] to include the effects of finite sample support used for covariance matrix estimation. However, the corresponding problem for non-Gaussian interference scenarios has received limited attention.

In this paper, we derive the NHD for non-Gaussian interference scenarios, which can be modeled by spherically invariant random processes (SIRP) and present a statistical analysis of the resultant NHD test statistic. In general, the problem of non-homogeneity

detection for SIRPs is quite difficult due to the fact that the underlying SIRP covariance matrix and characteristic PDF are unknown. For convenience, knowledge of the SIRP characteristic PDF is assumed in this paper.

3.2 Preliminaries

Let $\mathbf{x} = [x_1 \ x_2 \ \ldots x_M]^T$ denote a complex spherically invariant random vector (SIRV) having zero mean, positive definite Hermitian covariance matrix $\mathbf{R}$ and characteristic PDF $f_V(v)$. The PDF of $\mathbf{x}$ is given by [15]

$$f(x) = \pi^{-M}|\mathbf{R}|^{-1}h_{2M}(q) \tag{3.1}$$

where $|.|$ denotes determinant and

$$\begin{aligned} q &= \mathbf{x}^H\mathbf{R}^{-1}\mathbf{x} \\ h_{2M}(w) &= \textstyle\int_0^\infty v^{-2M} exp(-\frac{w}{v^2}) f_V(v) dv. \end{aligned} \tag{3.2}$$

Every SIRV admits a representation of the form [16] $\mathbf{x} = \mathbf{z}V$, where $\mathbf{z}$ has a complex-Gaussian PDF, $CN(0, \mathbf{R})$, and V is a statistically independent random variable with PDF $f_V(v)$. In practice, $\mathbf{R}$ and $f_V(v)$ are unknown. For the purpose of this paper, we assume knowledge of $f_V(v)$ and treat the problem of non-homogeneity detection with respect to unknown $\mathbf{R}$.

Previous work [1–4, 8, 12–14] employed the GIP based NHD for Gaussian interference scenarios. However, the GIP based method relying on the statistics of $Q = \mathbf{x}^H\hat{\mathbf{R}}^{-1}\mathbf{x}$ is unsuitable for SIRV interference scenrios. This is due to the fact that the covariance matrix estimate for this problem can be obtained to within a constant of the covariance matrix underlying the Gaussian component of the SIRV. Typically, this constant is unknown in practice. Consequently, the PDF of Q, its moments, and the threshold setting for the goodness-of-fit test proposed in [13] cannot be determined. Consequently, we seek a test statistic, which is invariant to the unknown scaling.

3.3 Nonhomogeneity Detector for Non-Gaussian Interference Scenarios

Let $\mathbf{x} \sim SIRV[0, \mathbf{R}_T, f_V(v)]$ denote the complex SIRV test data vector, where $\mathbf{R}_T$ is unknown. Further, $\mathbf{x}_i$, $i = 1, 2, \ldots K \sim SIRV[0, \mathbf{R}_T, f_V(v)]$ denote iid target free training data. For homogeneous training data, $\mathbf{R}_T = \mathbf{R}_x = \mathbf{R}$. The first step in deriving the NHD detector for SIRVs involves obtaining the maximum likelihood estimate of the underlying covariance matrix. This estimate is then used in a test statistic which exhibits maximal invariance with respect to the unknown scaling of the estimated covariance matrix. The resulting test statistic takes the form of a normalized adaptive matched filter (NAMF), which has been extensively analyzed in [17, 18] and references therein.

3.3.1 Covariance Matrix Estimation

The unknown covariance matrix is estimated from target free training data consisting of independent identically distributed SIRVs sharing the covariance matrix of the noise in the test cell. Maximum likelihood (ML) estimation of the covariance matrix for SIRVs was first considered in [19]. The work of [19] showed that covariance matrix estimation for SIRVs can be treated in the framework of a complete-incomplete data problem and pointed out that the maximum likelihood estimate of the covariance matrix is a weighted sample matrix. Since the problem does not permit a closed form solution, [19, 20] uses an iterative method known as the expectation-maximization (EM) algorithm. More precisely, let $\mathbf{x}_i$, $i = 1, 2, \ldots, K$ denote independent identically distributed training data sharing the covariance matrix of the test data vector $\mathbf{x}$. The work of [19, 20] shows that the ML estimate of the covariance matrix is given by

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{i=1}^{K} c_i \mathbf{x}_i \mathbf{x}_i^H \tag{3.3}$$

where

$$\begin{aligned} c_i &= -\frac{h'_{2M}(q_i)}{h_{2M}(q_i)} \\ h'_{2M}(w) &= \frac{\partial h_{2M}(w)}{\partial w} = -h_{2M+2}(w) \end{aligned} \tag{3.4}$$

and $q_i = \mathbf{x}_i^H \hat{\mathbf{R}}^{-1} \mathbf{x}_i$, $i = 1, 2, \ldots K$. Clearly the transcendental nature of the estimate precludes obtaining a closed form solution. Consequently, [19] used the EM algorithm to obtain an iterative solution to the problem. We adopt the approach of [19] for obtaining the covariance matrix estimate in this work. It can be readily shown that the EM algorithm at convergence produces an estimate which is to within a multiplicative constant of the covariance matrix estimate of the Gaussian component underlying the SIRV. Details pertaining to the initial start and convergence properties of the EM algorithm can be found in [19]. The next step is to use this estimate in a maximally invariant decision statistic for non-homogeneity detection.

3.3.2 Maximally Invariant NHD Test Statistic

The maximal invariant statistic for different scaling of test and training data is given by [17]

$$\Lambda_{NAMF} = \frac{|\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{x}|^2}{[\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{s}][\mathbf{x}^H \hat{\mathbf{R}}^{-1} \mathbf{x}]} \tag{3.5}$$

where $s = \frac{1}{\sqrt{M}}[1 \;\; 1 \;\; \ldots 1]^T$. Invariance properties of the test statistic of eq (3.5) and its geometrical representation have been studied in [17] and references therein for the case of Gaussian interference statistics using a sample covariance matrix estimate. In SIRP interference, however, each training data vector realization is scaled by a different realization of V. Consequently, maximal invariance of the test statistic of eq (3.5) afforded by the sample covariance matrix estimate no longer applies. This is due to the fact that the

sample covariance matrix is no longer the maximum likelihood estimate of the covariance matrix for SIRV scenarios. However, using an estimated covariance matrix of the form of eq (3.3) restores the maximal invariance property of the test statistic of eq (3.5). This is due to the fact that the resultant covariance matrix estimate is to within a multiplicative constant of the covariance matrix corresponding to the Gaussian component of the SIRV.

3.3.3 PDF and Moments of the Non-Gaussian NHD Test Statistic

The PDF and moments of the NHD test statistic are readily determined in terms of the corresponding quantities of an equivalent random variable defined by

$$\Lambda_{eq} = \frac{\Lambda_{NAMF}}{1 - \Lambda_{NAMF}} \tag{3.6}$$

It has been shown in [17, 21, 22] that Λ_{eq} admits a representation as the ratio of an F-distributed random variable and a beta-distributed loss factor. In this effort, we are interested in the PDF of Λ_{NAMF} under the condition where no target is present in the test data vector $\mathbf{x}$. Specifically, it can be shown from the work of [17,21,22] that the PDF of Λ_{NAMF} is given by

$$f_{\Lambda_{NAMF}}(r) = \int_0^1 \frac{L(1-\gamma) f_\Gamma(\gamma)(1-r)^{L-1} d\gamma}{[1-(1-\gamma)r]^{L+1}} \tag{3.7}$$

where $L = K - M + 1$ and Γ is the loss factor random variable, whose PDF is given by

$$f_\Gamma(\gamma) = \frac{1}{\beta(L+1, M-1)} \gamma^L (1-\gamma)^{M-2}. \tag{3.8}$$

The mean of Λ_{NAMF} is somewhat difficult to calculate. Consequently, we work with the mean of Λ_{eq} given by

$$E(\Lambda_{eq}) = \frac{K}{(K-M)(M-2)}. \tag{3.9}$$

The statistical equivalence of Λ_{eq} to within a scalar of the ratio of a F-distributed random variable and a beta-distributed loss factor in that it permits rapid calculation of the moments of Λ_{eq}. More importantly, it is extremely useful in Monte-Carlo studies involving simulation of Λ_{NAMF}. For homogeneous training data, the use of (3.6) circumvents the need to explicitly generate the test data vector $\mathbf{x}$ and the training data vectors used for covariance estimation. For large M and perforce K, significant computational savings can be realized from the method of (3.6). It is instructive to note that the PDF of Λ_{NAMF} as well as its mean depend only on M and K, which are under the control of a system designer, and not on nuisance paramters such as the true covariance matrix underlying the interference scenario. Furthermore, for $K \to \infty$ the mean of eq (3.9) converges to $E(\Lambda_{eq}) = \frac{1}{(M-2)}$, corresponding to the mean of an F-distributed random variable.

3.3.4 Goodness-of-Fit Tests

Since the PDF and mean of Λ_{NAMF} are known, a formal goodness of fit test can be used for non-homogeneity detection in non-Gaussian interference scenarios. In particular, we form empirical realizations of Λ_{NAMF} from each training data realization using a moving window approach. In this approach each training data vector is treated as a test cell data vector, whose covariance matrix is estimated from neighboring cell data according to eq (3.3). We then test for statistical consistency of these realizations of Λ_{NAMF} with the PDF of eq (3.7). For this purpose a convenient type-I error (typically between 0.01 and 0.1) given by

$$P_e = \int_0^1 \frac{f_\Gamma(\gamma)}{[1+(1-\gamma)\eta]^L} d\gamma \tag{3.10}$$

is chosen. The threshold, η, is determined by a numerical inversion of eq (3.10). Realizations of Λ_{NAMF}, which exceed η correspond to nonhomogeneous training data. A second test for training data nonhomogeneity is based on comparing each realization of Λ_{eq} with its theoretically predicted mean given by eq (3.9) and retaining those realizations which exhibit the least deviation. Performance analysis of these NHD methods is presented in the next section.

3.4 Performance Analysis

Performance of the goodness-of-fit test with simulated and measured data is presented here. Figure 1 shows the plot of the PDF of Λ_{NAMF} with K as a parameter. Observe that the variance of Λ_{NAMF} decreases with increasing K. Figure 2 shows a plot of the Type-I error versus the threshold, η, with K as a parameter. For a given type-I error, the threshold decreases with increasing K, in conformance with the results of Figure 1. Figure 3 shows the performance of the goodness-of-fit test for simulated homogeneous data from the K-distribution [15] with shape parameter 0.5 using the covariance estimate of eq (3.3). The results reveal the lack of nonhomogeneity in that no realization of Λ_{NAMF} exceeds the threshold. Figure 4 shows the performance of the goodness-of-fit test in non-homogeneous K-dsitributed clutter with shape parameter 0.5. Nonhomogeneity of the data is evident in those range bins where Λ_{NAMF} exceeds η. Figure 5 shows the results of the goodness of fit test for the MCARM data [23] using acquisition 220 on Flight 5, cycle e for 8 channels and 16 pulses. The NHD test statistic, Λ_{NAMF}, and the threshold, η, are plotted as a function of range. Statistical analysis of the data indicates that the data is well approximated by the Gaussian distribution. This fact considerably simplifies the analysis in that the covariance matrix estimate is simply the sample covariance matrix. Non-homogeneity of the training data is evident in those bins for which Λ_{NHD} exceeds η. Figure 6 shows the results from the selection procedure based on comparing the empirically formed Λ_{eq} with its theoretically predicted mean given by eq (3.9). The data set used for this example is identical to the data used in Figure 5. Relevant test parameters are reported in the plot. We observe a significant increase in the number of deviations from the thoretically

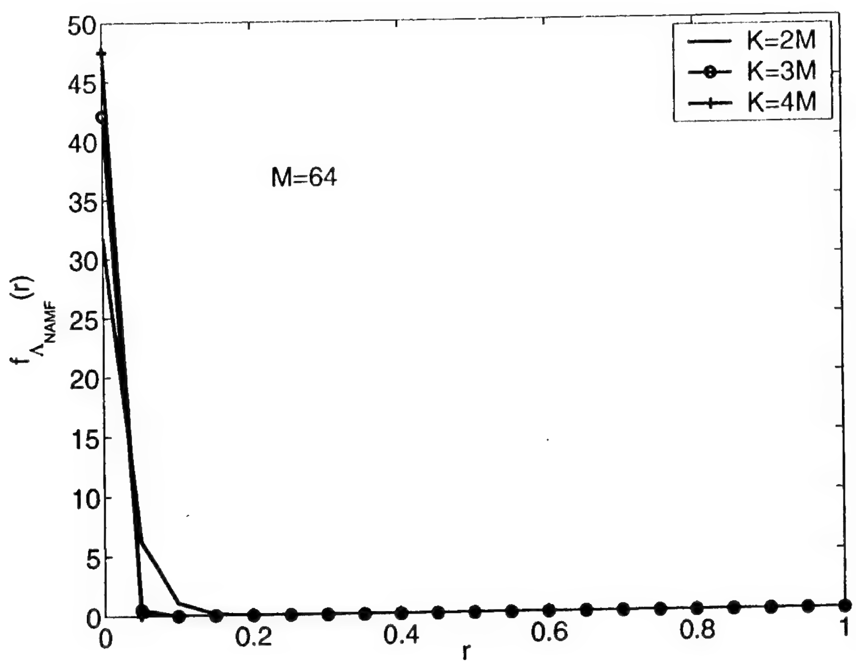

Figure 3.1: NHD Test Statistic PDF

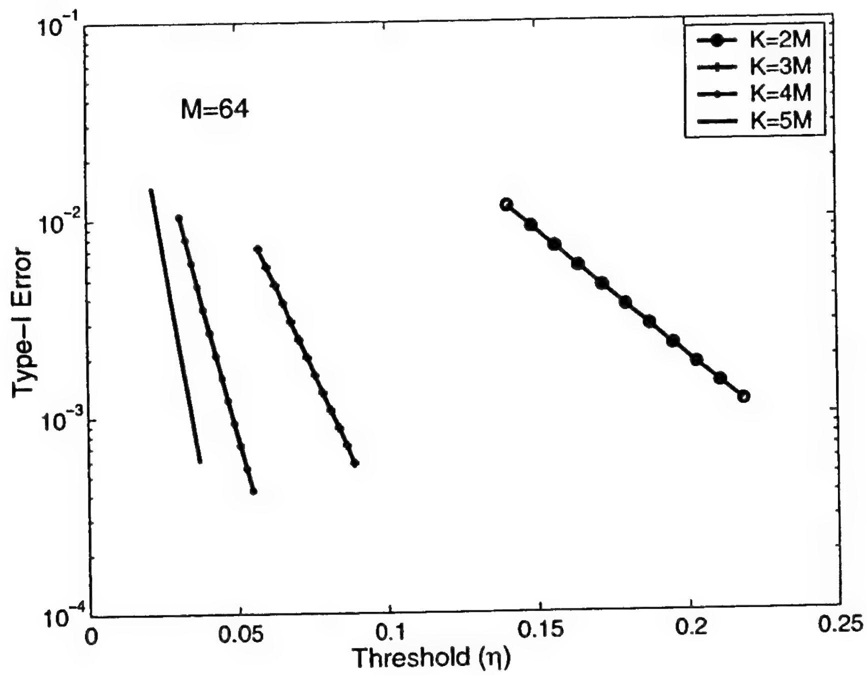

Figure 3.2: Type-I Error vs Threshold for NHD Test Statistic

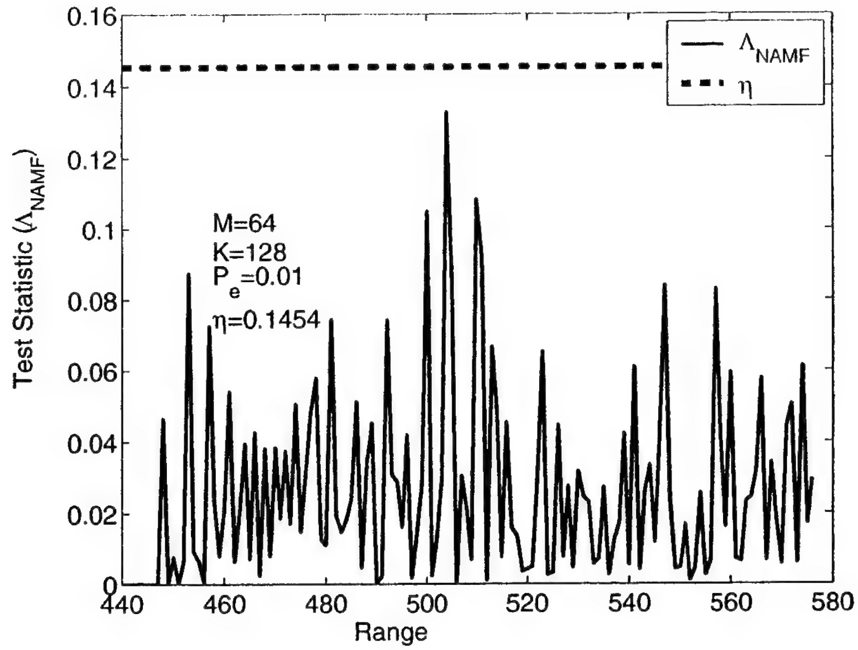

Figure 3.3: Type-I Error vs Threshold for NHD Test Statistic

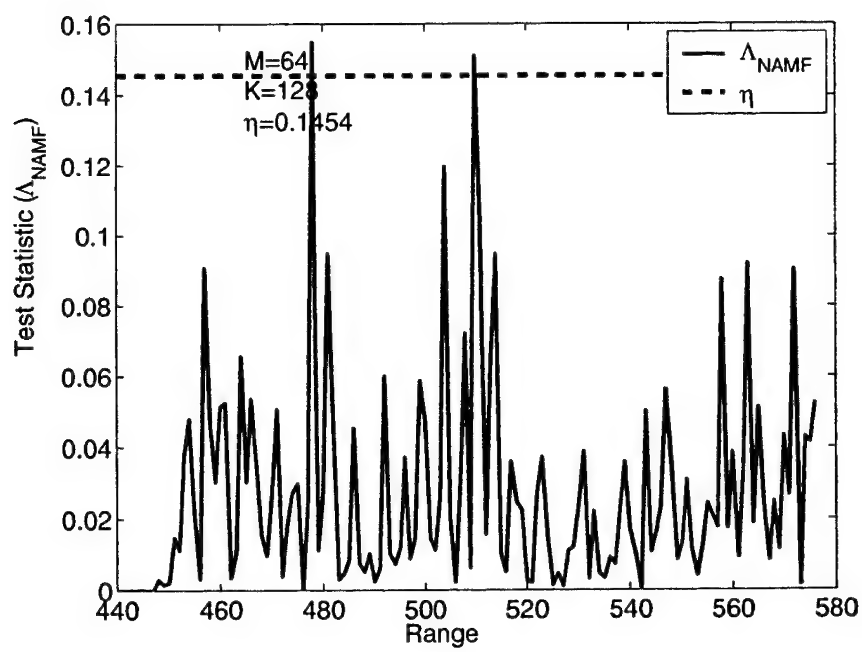

Figure 3.4: Type-I Error vs Threshold for NHD Test Statistic

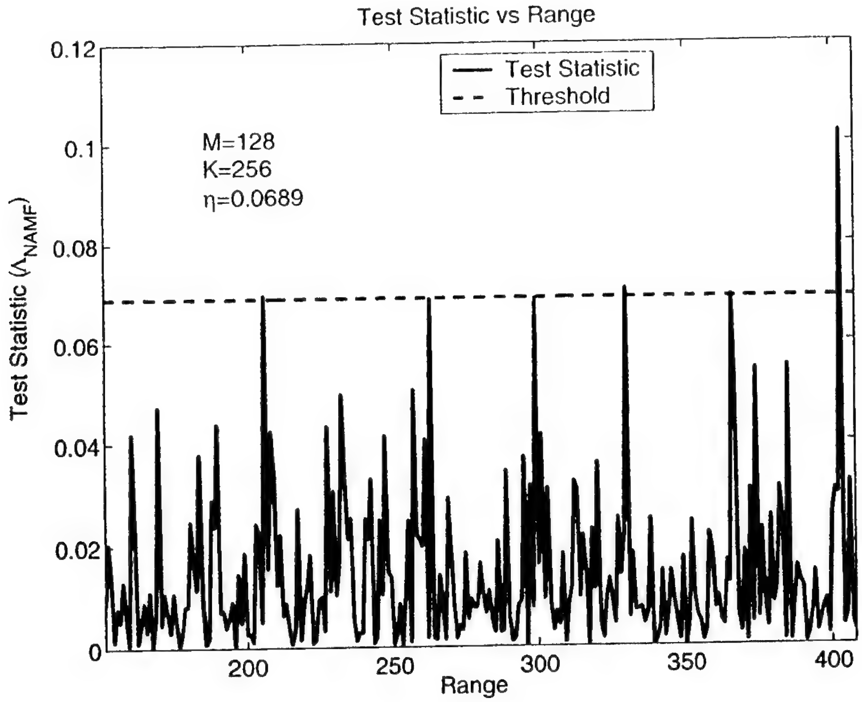

Figure 3.5: NHD Test Statistic vs Range

predicted mean given by eq (3.9). This is due to the fact that we are dealing with a limited number of realizations of the NHD test statistic.

3.5 Conclusion

This paper provides a rigorous statistical characterization of the NHD for non-Gaussian interference scenarios which can be modeled as a spherically invariant random process. It is noted that the NHD statistic admits a simple representation in terms of a ratio of an F distributed random variable and a beta distributed loss factor. A formal goodness-of-fit test based on this representation, which follows a randomized F-distribution, is derived. Performance analysis of the method is considered in some detail using measured data from the MCARM program. The illustrative examples validate the approach taken and confirm the results. Future work would include extensive performance analysis using simulated and measured data showing the resulting impact on STAP performance. The performance of several STAP algorithms in Gaussian and non-Gaussian interference scenarios has been considered in [18]. Future work will address performance of the methods treated in [18] with suitable NHD pre-processing.

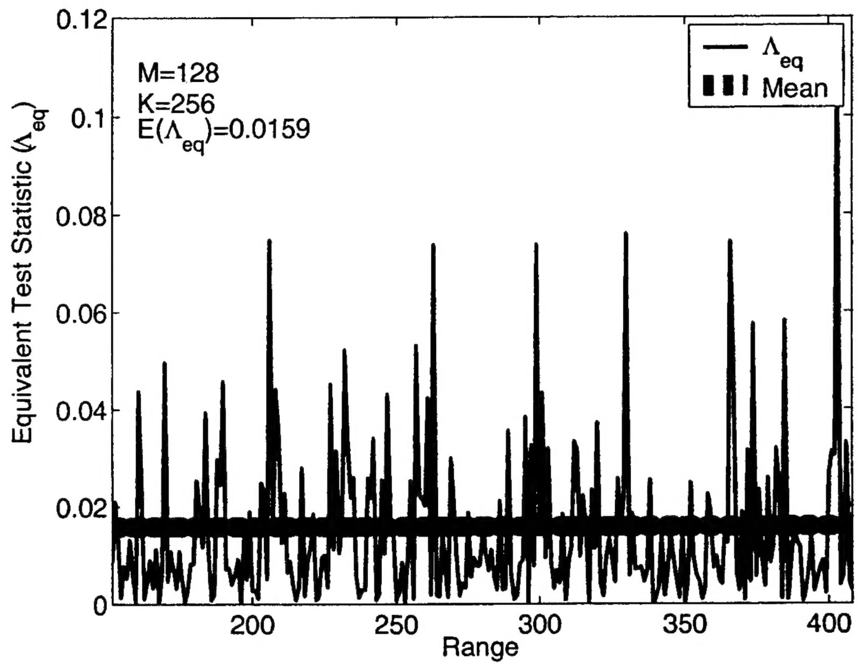

Figure 3.6: NHD Test Statistic vs Range

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